Reg. No. : | | | | | | | | | | | |

Question Paper Code : 63254

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2016.

First Semester

Civil Engineering

MA 1101 — MATHEMATICS — I

(Common to All Branches)

(Regulations 2008)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the sum of the Eigen values of the inverse of the matrix $A = \begin{bmatrix} 3 & 0 & 0 \\ 8 & 4 & 0 \\ 6 & 2 & 5 \end{bmatrix}$.

2. Find the characteristic equation of the matrix $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$.

3. If $\cos\alpha, \cos\beta, \cos\gamma$ are the direction cosines of any line prove that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.

4. Find the centre and radius of the sphere $x^2 + y^2 + z^2 + 6x - 2y + 4z + 5 = 0$.

5. Find the radius of curvature of the parabola $x^2 = 4ay$ at $x = 2a$.

6. If the centre of curvature of a curve at a variable point 'θ' is $\left(a\log\left(\cot\frac{\theta}{2}\right), \frac{a}{\sin\theta}\right)$ find the evolute.

7. State Euler's theorem for homogeneous functions.

8. If $u = f(x,y)$ then prove that $\frac{du}{dx} = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\frac{dy}{dx}$.

9. Find the particular integral of $(D^2 - 2D)\,y = e^x \cos x$.

10. Solve $(D^2 - 2D + 1)^2 y = 0$

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find all the Eigen values and Eigen Vectors of the matrix $A = \begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$.

(ii) Diagonalise the matrix $A = \begin{bmatrix} 2 & 2 & -7 \\ 2 & 1 & 2 \\ 0 & 1 & -3 \end{bmatrix}$ by similarity transformation.

Or

(b) (i) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ by using Cayley Hamilton theorem.

(ii) Obtain an orthogonal transformation, which will transform the quadratic form $2x_1^2 + 6x_2^2 + 2x_3^2 + 8x_1x_3$ into a canonical form.

12. (a) (i) Find the equation of the image of $\frac{x-1}{3} = \frac{y-3}{5} = \frac{z-4}{2}$ in the plane $2x - y + z + 3 = 0$. (8)

(ii) Find the length of the shortest distance between the pair lines $\frac{x-1}{1} = \frac{y-2}{-2} = \frac{z-3}{3}$ and $\frac{x+1}{2} = \frac{y}{-1} = \frac{z-1}{3}$. (8)

Or

(b) (i) Find the equation of tangent plane of the sphere $x^2 + y^2 + z^2 - 4x - 2y - 6z + 5 = 0$ which are parallel to the plane $x + 4y + 8z = 0$. Find also their point of contract. (8)

(ii) Find the equation of the cone whose vertex is (1,2,3) and guiding curve is the circle $x^2 + y^2 + z^2 = 4, x + y + z = 1$. (8)

13. (a) (i) Find the radius of curvature at 't' on $x = e^t \cos t$, $y = e^t \sin t$. (8)

(ii) Find the evolute of the parabola $y^2 = 4ax$. (8)

Or

(b) (i) Find the circle of curvature for the curve $y = 3x^3 + 2x^2 - 3$ at the point $(0, -3)$. (8)

(ii) Find the envelope of $\frac{x}{a} + \frac{y}{b} = 1$, where the parameters a, b are related by $a^2 + b^2 = c^2$, where c is known. (8)

14. (a) (i) Verify the Euler's theorem for $u = e^{x^2+y^2}$. (8)

(ii) Find the maximum and minimum values of $\sin x \sin y \sin(x+y)$. (8)

Or

(b) (i) If $z = f(x,y)$, where $x = u^2 - v^2, y = 2uv$, prove that $\left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\right) = 4(u^2 + v^2)\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right)$. (8)

(ii) Expand $x^4 + x^2y^2 - y^4$ as a Taylor series expansion about the point (1,1) upto third order terms. (8)

15. (a) (i) Using method of variation of parameters solve the following differential equation $y'' - 2y' + y = xe^x$. (8)

(ii) Solve $\left[(x+1)^2 D^2 + (x+1)D + 1\right]y = (x+1)^3 \log(x+1)$. (8)

Or

(b) (i) Solve the system of differential equations $\frac{dx}{dt} + 5x - 2y = t$; $\frac{dy}{dt} + 2x + y = 0$. (8)

(ii) Solve $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = x^2 \sin(\log x)$. (8)

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